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ESTIMATION OF MEAN IN CASE OF MEASUREMENT ERRORS USING LOGARITHMIC TYPE ESTIMATOR

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ABSTRACT

This study deals the problem of mean estimation in presence of measurement errors (ME) using logarithmic estimator in simple random sampling (SRS). The expression of mean square error of the suggested estimator is determined to the approximation of order first. The suggested estimator is compared with usual mean estimator, classical ratio and product estimators and the efficiency conditions are obtained to show efficacy of the proposed estimator over the conventional estimators. Subsequently, to enhance the theoretical findings, numerical and simulation studies are also performed.

Keywords: Mean square error, Measurement errors, Simulation study.

Mathematical subject classification: 62D05

1. INTRODUCTION

In survey methodology, the primary aim of researchers is to increase the accuracy of their estimation procedures such as ratio, product, regression and logarithmic methods. These estimation procedures efficiently use the information on auxiliary variable that is strongly correlated with the study variable either at estimation stage or at designing stage or at both stages. The method of ratio estimation considers the auxiliary information that is correlated positively with the study variable to enhance the efficiency which results in the improved estimators when the regression line of Y on X is linear and passes through origin, see, Cochran (1940). The conditions may also exist when the regression line of Y on X is linear but it does not pass through the origin. Under such conditions, it is more appropriate to consider the regression type estimator to compute the population means, see, Cochran (1977). When the study and auxiliary variables are correlated negatively then the product method of estimation provide efficient results, see, Murthy (1964). Further, when logarithmic relationship arises between study and auxiliary variables then in such situations the logarithmic estimators would perform fare better than the other conventional estimators, see, Bhushan and Kumar (2020, 2021).

Further, in survey methodology, it is presumed that the data collected on study variable Y and auxiliary variable X are the actual measurement of observations. But, in practice, this assumption violates due to many reasons and may be recorded with some kind of errors known as ME. The ME is defined as the difference between the observed and true values of the parameters. Cochran (1968) and Murthy (1967) examined the impact of ME in survey sampling. The effect of ME meld with data on the statistical properties of estimators of parameters is described in the text books by various authors namely Fuller (1987), Cheng and Van Ness (1994) and Carroll *et al.* (2006). Biemer and Stokes (1991) described that the existence of ME can result in serious misleading inference. Further, in literature of survey sampling, estimation of parameters by using auxiliary variable is extensive and significant. The ratio, regression and product estimators are considered in computation of parameters by various researchers. Shalabh (1997), Manisha and Singh (2001), Sahoo *et al.* (2006), Gregoire and Salas (2009) and Kumar *et al.* (2011) have examined the impact of ME using ratio and regression estimators for the estimation of population mean. The objective of current paper is to address the mean estimation procedure using logarithmic type estimator in presence of ME under SRS.

Let a finite population of size N from that a size n sample is quantified utilizing simple random sampling without replacement. We consider the case when data values may be recorded with ME. Let the observed values be denoted by (y_1, z_1); $i = 1, 2, ..., n$ and the true values be denoted by $(Y_i - Z_i)$. Let the observed values be expressible in additive forms as $y_i = Y_i + U_i$ and $z_1 = Z_i + V_i$ such that $U \sim n\,(0, \sigma_U^2)$ and $V \sim N\,(0, \sigma_v^2)$. It is presumed that the error variables U and V are uncorrelated to each other as well as uncorrelated to other combinations of X and Y, respectively. Let μ_Y, μ_Z be the population means and $\mu_Y^2\, \mu_Z^2$ be the population variance of study and auxiliary variables, respectively. In case of ME $s_Z^2 = (n-1)^{(-1)} \sum_{(i=1)}^{n} (z_i - \overline{z})^2$ and $s_y^2 = (n-1)^{-1} \sum_{i=1}^{n} (y_i - \overline{y})^2$ are not unbiased estimators of the population variance σ_Y^2, σ_Z^2 respectively. Therefore, the expected the expected values of

s_z^2 and s_y^2 in the presence of *ME* are given by $E(s_z^2) + \sigma_Z^2 + \sigma_V^2$ and $E(\sigma_y^2 = \sigma_Y^2 + \sigma_U^2$.

To derive mean square error (MSE) of suggested estimator in case of *ME*, we assume that $\bar{y} = \mu_y(1 + e_0)$ and $\bar{z} = \mu_z(1 + e_1)$ such that $E(e_0) = 0$, $E(e_1) = 0$, $E(e_0^2) = \dfrac{C_Y^2}{n\zeta_y}$, $E(e_1^2) = \dfrac{C_Z^2}{n\zeta_z}$ and $E(e_0 e_1) = \dfrac{\rho C_Y C_Z}{n}$.

where $C_Y = \dfrac{S_Y}{\mu_Y}$ and $C_Z = \dfrac{S_Z}{\mu_Z}$ are the population coefficient of variations for study and auxiliary variables, respectively, ρ is the population correlation coefficient between study and auxiliary variables. Also, $\zeta_Y = \sigma_Y^2/(\sigma_Y^2 + \sigma_U^2)$ and $\xi_Z = \sigma_Z^2/(\sigma_Z^2 + \sigma_V^2)$ are the reliability ratio of the study and auxiliary variables that lies between 0 and 1.

The variance of usual mean estimator $\bar{y}$ in the presence of ME is given by

$$V(\bar{y}) = \frac{\sigma_Y^2}{n} + \frac{\sigma_U^2}{n} \tag{1.1}$$

Shalabh (1997) developed the conventional ratio and product estimators in case of ME using SRS as

$$\bar{y}_r = \bar{y}\left(\frac{\mu_Z}{\bar{z}}\right) \tag{1.2}$$

$$\bar{y}_p = \bar{y}\left(\frac{\bar{z}}{\mu_Z}\right) \tag{1.3}$$

The MSE of the estimators t_r and t_p is given by

$$MSE(\bar{y}_r) = \mu_Y^2\left[\frac{C_Y^2}{n} + \frac{C_Y^2}{n} - \frac{2\rho C_Z C_Y}{n}\right] + \mu_Y^2\left[\frac{C_Y^2}{n}\frac{\sigma_U^2}{\sigma_Y^2} + \frac{C_Z^2}{n}\frac{\sigma_V^2}{\sigma_Z^2}\right] \tag{1.4}$$

$$MSE(\bar{y}_p) = \mu_Y^2\left[\frac{C_Y^2}{n} + \frac{C_Y^2}{n} - \frac{2\rho C_Z C_Y}{n}\right] + \mu_Y^2\left[\frac{C_Y^2}{n}\frac{\sigma_U^2}{\sigma_Y^2} + \frac{C_Z^2}{n}\frac{\sigma_V^2}{\sigma_Z^2}\right] \tag{1.5}$$

wwhere the first term in (1.4) and (1.5) represent the MSE of $\bar{y}_r$ and $\bar{y}_p$ estimators without ME whereas the last terms of the expressions of (1.4) and (1.5) represent the contribution of ME. The remainder of the article is divided in few sections. The suggested logarithmic type estimator of population mean and its properties under ME are given in section 2. In section 3, the efficiency conditions are obtained by comparing the MSE expressions of the suggested and conventional estimators. The numerical and simulation studies are performed in section 4 and section 5, respectively. The discussion of empirical results is given in section 6 and conclusion is presented in section 7.

2. SUGGESTED ESTIMATORS

The objective of current article is to suggest an efficient class of estimator for the computation of population mean in case of ME. Motivated by Bhushan and Kumar (2020, 2021), we suggest a logarithmic type estimator in presence of ME under SRS as

$$T = \bar{y} + \alpha \log\{1 + (\bar{Z} - \bar{z})\} \tag{2.1}$$

where α is duly opted scalar to be determine.

Theorem 2.1. The minimum MSE of the suggested estimator T is given by.

$$min.\ MSE(T) = \frac{\mu_Y^2}{n}\frac{C_Y^2}{\zeta_y}[1 - \rho^2 \zeta_y \zeta_z] \tag{2.2}$$

Proof. We express the suggested estimator T in error terms as

$$T - \mu_Y = \mu_Y\left(e_0 - \frac{\alpha}{R}e_1 - \frac{\alpha}{R}\frac{Z}{2}e_1^2\right) \tag{2.3}$$

Squaring and taking expectation on both sides of (2.3), we get the $MSE(T)$ to the first order of approximation as

$$MSE(T) = \frac{\mu_Y^2}{n}\left[\frac{C_Y^2}{\zeta_y} + \alpha^2\frac{C_Z^2}{\zeta_z} - 2\alpha\rho C_Y C_Z\right] \tag{2.4}$$

Minimizing (2.4) with respect to α, we get

$$\alpha_{(opt)} = \rho\zeta_z\frac{C_Y}{C_Z} \tag{2.5}$$

Now, putting the value of $\alpha_{(opt)}$ in (2.4), we get

$$\min.MSE(T) = \frac{\mu_Y^2}{n}\frac{C_Y^2}{\zeta_Y}[1-\rho^2\zeta_y\zeta_z] \tag{2.6}$$

3. EFFICIENCY CONDITIONS

In present section, we examine theoretically the execution of the proposed estimator with the existing estimators. We compare the minimum MSE of suggested estimators with the minimum MSE of existing estimators and obtained the following efficiency conditions.

I. From (1.1) and (2.6), we get

$$MSE(\bar{y}) > MSE(T) \Rightarrow \rho^2\zeta_y\zeta_z > 0 \tag{3.1}$$

II. From (1.4) and (2.6), we get

$$MSE(\bar{y}_r) > MSE(T) \Rightarrow \rho^2 > \frac{2\rho C_Z}{\zeta_z C_Y} - \frac{C_Z^2}{\zeta_y^2 C_Y^2} \tag{3.2}$$

III. From (1.5) and (2.6), we get

$$MSE(\bar{y}_p) > MSE(T) \Rightarrow \rho^2 > -\frac{2\rho C_Z}{\zeta_z C_Y} - \frac{C_Z^2}{\zeta_y^2 C_Y^2} \tag{3.3}$$

The suggested estimator performs better than the conventional estimators under the above conditions. Further, these conditions are verified empirically using natural and artificial data sets.

4. NUMERICAL STUDY

We illustrate the performance of the proffered estimator using two real populations. The data of population 1 and population 2 are taken from Gujarati and Sangeetha (2007) and the book of U.S. Census Bureau (1986) respectively, and presented in Table 1 for quick review. Now, the descriptive statistics summarized in Table 1 is used to compute the percent relative efficiency (PRE) of the suggested estimator regarding the usual mean estimator, classical

Table 1: Descriptive statistics of real populations

Descriptive statistics	N	n	μ_x	μ_y	σ_z^2	σ_y^2	ρ	σ_U^2	σ_V^2
Population 1	10	4	170	127	3300	1278	0.964	36	36
Population 2	56	15	75.59	61.59	155.5	577.44	–0.508	16	16

ratio estimator $\bar{y}_r$ and product estimator $\bar{y}_p$ using the following expressions.

$$E_1 = \frac{MSE(\bar{y}_m)}{MSE(T)} \times 100 \tag{4.1}$$

$$E_2 = \frac{MSE(\bar{y}_r)}{MSE(T)} \times 100 \tag{4.2}$$

$$E_3 = \frac{MSE(\bar{y}_p)}{MSE(T)} \times 100 \tag{4.3}$$

The results of numerical study for population1 and population 2 are cited in Table 2.

Table 2: PRE of different estimators based on real populations

	Population 1		*Population 2*	
	PRE without ME	*PRE with ME*	*PRE without ME*	*PRE with ME*
E_1	1414.347	977.128	134.783	129.480
E_2	179.078	142.134	265.500	205.866
E_3	6726.034	4392.721	101.004	100.905

5. SIMULATION STUDY

This section presents a simulation study to generalize the findings of numerical study accomplished in previous section. We generate a normal population of size $N = 2000$ from R software using a multivariate normal distribution consisting mean vector $(\mu_Y, \mu_X, 0, 0)$ and covariance matrix

$$\begin{pmatrix} \sigma_Y^2 & \rho\sigma_z\sigma_Y & 0 & 0 \\ \rho\sigma_z\sigma_Y & \sigma_Z^2 & 0 & 0 \\ 0 & 0 & \sigma_U^2 & 0 \\ 0 & 0 & 0 & \sigma_V^2 \end{pmatrix}$$

where $\mu_Y = 25$, $\mu_X = 30$, $\sigma_Z^2 = (15, 20)$, $\sigma_Y^2 = (15, 20)$, $\sigma_U^2 = (3, 5)$, $\sigma_Y^2 = (3, 5)$, $\rho = (-0.9, -0.5, -0.1, 0.1, 0.5, 0.9)$.

We have drawn random samples of size $n = 100$ and $n = 200$ from this population. We have considered 40000 iterations to compute PRE of the suggested estimator regarding the usual mean estimator, classical ratio estimator and classical product estimator using the following expressions.

$$E_1 = \frac{\frac{1}{40000}\sum_{i=1}^{40000}(\bar{y}_m - \bar{Y})^2}{\frac{1}{40000}\sum_{i=1}^{40000}(T - \bar{Y})^2} \times 100 \tag{5.1}$$

$$E_2 = \frac{\frac{1}{40000}\sum_{i=1}^{40000}(\bar{y}_r - \bar{Y})^2}{\frac{1}{40000}\sum_{i=1}^{40000}(T - \bar{Y})^2} \times 100 \tag{5.2}$$

$$E_3 = \frac{\frac{1}{40000}\sum_{i=1}^{40000}(\bar{y}_p - \bar{Y})^2}{\frac{1}{40000}\sum_{i=1}^{40000}(T - \bar{Y})^2} \times 100 \tag{5.3}$$

The PRE of usual mean estimators, classical ratio and product estimators are calculated for different values of parameters σ_Z^2, σ_Y^2, σ_U^2, σ_V^2, and ρ by using (5.1), (5.2) and (5.3) respectively. The results are presented from Table 3 to Table 6.

6. DISCUSSION OF NUMERICAL AND SIMULATION RESULTS

After profoundly analysing the numerical and simulation results cited from Table 2 to Table 6, we draw the following observations.

i. From the results of numerical study for population 1 presented in Table 2, it is seen that the PRE of the suggested estimator regarding conventional estimators such as: usual mean estimator remain 1414.347 without ME and 944.128 with ME; classical ratio estimator remain 179.078 without ME and 142.134 with ME; classical product estimator remain 6726.034 without ME and 4392.731 with ME. Proceeding in the same manner, it is also observed from the results of population 2 that the PRE of the suggested estimator regarding the existing estimators is greater than 100 in presence and absence of ME which shows the dominance of the suggested estimator over the existing estimators. Moreover, it is mentioned that the PRE decreases in presence of ME in both the populations.

ii. From the simulation results presented in Table 3 for $\sigma_Z^2 = 15$, $\sigma_Y^2 = 15$, n = 100 and 200 it is observed that when $\sigma_U^2 = 3$ and $\sigma_V^2 = 3$:

   a. The PRE of the proposed estimator regarding the usual mean estimator is greater than 100 for each value of correlation coefficient

Table 3: PRE of different estimators when $\sigma_Z^2 = 15$ and $\sigma_y^2 = 15$

			$n = 100$			$n = 200$		
σ_U^2	σ_V^2	ρ	E_1	E_2	E_3	E_1	E_2	E_3
3	3	–0.9	231.557	675.691	101.688	231.172	674.366	101.410
		–0.5	125.257	306.292	120.222	124.879	305.419	119.794
		–0.1	102.217	195.060	153.037	101.877	194.474	152.493
		0.1	102.218	151.978	193.670	101.878	151.364	192.969
		0.5	125.262	119.509	304.129	124.884	119.072	303.139
		0.9	231.557	101.516	674.172	231.172	101.301	673.043
3	5	–0.9	206.577	617.180	105.100	206.176	615.779	104.775
		–0.5	122.111	308.488	127.092	121.770	307.671	126.668
		–0.1	101.983	203.003	161.077	101.679	202.459	160.559
		0.1	101.982	159.938	201.533	101.678	159.346	200.870
		0.5	122.096	126.279	306.233	121.754	125.844	305.298
		0.9	206.577	104.862	615.740	206.176	104.609	614.530
5	3	–0.9	206.524	575.950	101.902	206.218	574.968	101.655
		–0.5	122.056	280.185	117.676	121.731	279.453	117.298
		–0.1	101.980	184.871	147.360	101.676	184.355	146.874
		0.1	101.983	146.456	183.704	101.679	145.909	183.089
		0.5	122.083	117.075	278.461	121.756	116.685	277.628
		0.9	206.524	101.742	574.677	206.218	101.550	573.861
5	5	–0.9	186.443	519.478	104.136	186.152	518.506	103.875
		–0.5	119.363	282.582	123.726	119.072	281.938	123.363
		–0.1	101.769	191.926	154.509	101.499	191.472	154.065
		0.1	101.770	153.538	190.692	101.499	153.031	190.137
		0.5	119.368	123.034	280.763	119.076	122.663	280.027
		0.9	186.443	103.944	518.310	186.152	103.740	517.492

ρ which is higher for highly positive and negative correlated values of correlation coefficient ρ.

b. The PRE of the proposed estimator regarding the classical ratio estimator is greater than 100 for each value of correlation coefficient which is higher for negatively correlated values of correlation coefficient and decreases as the varies from –0.9 to +0.9.

c. The PRE of the proposed estimator regarding the classical product estimator is greater than 100 for each value of correlation coefficient which is higher for positively correlated values of *r* and increases as the ρ varies from –0.9 to +0.9.

Table 4: PRE of different estimators when $\sigma_Z^2 = 15$ and $\sigma_y^2 = 20$

			$n = 100$			$n = 200$		
σ_U^2	σ_V^2	ρ	E_1	E_2	E_3	E_1	E_2	E_3
3	3	−0.9	245.649	651.746	100.959	245.187	650.322	100.677
		−0.5	126.674	279.761	110.881	126.281	278.921	110.457
		−0.1	101.989	170.707	138.160	101.627	170.116	137.571
		0.1	101.836	138.471	168.856	101.463	137.915	168.138
		0.5	126.454	110.508	277.210	126.043	110.096	276.224
		0.9	245.649	100.886	650.364	245.187	100.656	649.130
3	5	−0.9	216.642	586.616	100.867	216.173	585.149	100.546
		−0.5	123.294	280.065	115.691	122.940	279.281	115.274
		−0.1	101.772	176.700	144.222	101.449	176.150	143.662
		0.1	101.635	144.513	174.838	101.303	143.986	174.161
		0.5	123.085	115.249	277.508	122.714	114.844	276.584
		0.9	216.642	100.731	585.325	216.173	100.470	584.040
5	3	−0.9	220.767	566.450	100.644	220.385	565.329	100.386
		−0.5	123.943	261.381	109.779	123.594	260.649	109.394
		−0.1	101.825	164.824	134.991	101.492	164.288	134.451
		0.1	101.683	135.290	163.154	101.342	134.783	162.505
		0.5	123.767	109.456	259.222	123.401	109.079	258.363
		0.9	220.767	100.567	565.264	220.385	100.355	564.308
5	5	−0.9	197.898	508.416	100.728	197.540	507.337	100.459
		−0.5	120.960	262.021	114.118	120.648	261.370	113.748
		−0.1	101.624	170.292	140.527	101.329	169.810	140.026
		0.1	101.499	140.812	168.616	101.195	140.343	168.019
		0.5	120.788	113.731	259.842	120.461	113.372	259.072
		0.9	197.898	100.614	507.333	197.540	100.395	506.406

d. The similar inclination can be observed when when $\sigma_U^2 = 3$, $\sigma_V^2 = 15$, $\sigma_U^2 = 5$, $\sigma_V^2 = 3$, and $\sigma_U^2 = 5$, $\sigma_V^2 = 5$.

iii. The conclusion like point (ii) can also be drawn from table 4 based $\sigma_Z^2 = 15$, $\sigma_Y^2 = 20$, Table 5 based on $\sigma_Z^2 = 20$, $\sigma_Y^2 = 15$ and Table 6 based on $\sigma_Z^2 = 20$, $\sigma_Y^2 = 20$.

7. CONCLUSION

In this study, we introduced a logarithmic type estimator for the computation of population mean in case of ME under SRS and obtained its MSE expression. The efficiency conditions are obtained and exemplified using natural and artificial data sets. The numerical and simulation results

Table 5: PRE of different estimators when $\sigma_Z^2 = 20$ and $\sigma_y^2 = 15$

			n = 100			*n* = 200		
σ^2_U	σ^2_V	ρ	E_1	E_2	E_3	E_1	E_2	E_3
3	3	–0.9	244.038	808.759	107.588	243.576	807.103	107.257
		–0.5	126.670	347.990	131.285	126.278	346.853	130.760
		–0.1	101.842	219.840	178.223	101.465	219.054	177.625
		0.1	101.842	177.712	219.249	101.465	177.165	218.506
		0.5	126.442	132.306	349.566	126.032	131.898	348.619
		0.9	244.499	106.826	804.932	244.116	106.600	803.661
3	5	–0.9	220.903	747.457	112.759	220.447	745.792	112.398
		–0.5	123.965	350.525	138.449	123.605	349.448	137.928
		–0.1	101.689	228.160	186.605	101.344	227.412	186.033
		0.1	101.689	186.043	227.517	101.344	185.527	226.819
		0.5	123.777	139.569	352.250	123.404	139.173	351.374
		0.9	221.077	111.873	743.104	220.692	111.618	741.795
5	3	–0.9	216.361	682.404	107.166	216.026	681.298	106.893
		–0.5	123.256	316.268	127.307	122.918	315.320	126.843
		–0.1	101.638	206.634	169.608	101.303	205.951	169.085
		0.1	101.638	169.159	206.113	101.303	168.679	205.467
		0.5	123.035	128.168	317.463	122.684	127.810	316.679
		0.9	216.867	106.526	679.934	216.576	106.333	679.067
5	5	–0.9	196.620	617.417	110.197	196.297	616.305	109.912
		–0.5	120.943	318.936	133.600	120.639	318.099	133.161
		–0.1	101.502	213.987	177.027	101.195	213.366	176.549
		0.1	101.502	176.527	213.416	101.195	176.098	212.838
		0.5	120.763	134.553	320.272	120.445	134.221	319.598
		0.9	196.937	109.506	614.886	196.665	109.305	614.030

are reported from Table 2 to Table 6 by PRE. After the perusal of theoretical, numerical and simulation results, it is clear that the suggested estimator is rewarding in terms of PRE over the existing estimators. Thus, our proposed estimator is justified and recommended to the survey practitioners for the computation of population mean in case of ME.

Furthermore, the proposed estimator can also be examined for the estimation of population mean in case of ME under stratified simple random sampling.

Table 6: PRE of different estimators when $\sigma_Z^2 = 20$ and $\sigma_y^2 = 20$

			$n = 100$			$n = 200$		
σ^2_U	σ^2_V	ρ	E_1	E_2	E_3	E_1	E_2	E_3
3	3	–0.9	261.004	779.871	101.170	260.450	778.089	100.835
		–0.5	128.246	318.098	118.837	127.821	317.072	118.359
		–0.1	102.431	196.537	152.463	102.058	195.874	151.859
		0.1	102.431	152.065	196.033	102.058	151.519	195.431
		0.5	128.251	118.083	315.691	127.825	117.606	314.549
		0.9	260.907	100.847	774.587	260.443	100.617	773.199
3	5	–0.9	233.673	710.936	103.307	233.140	709.191	102.952
		–0.5	125.345	318.869	124.117	124.957	317.907	123.645
		–0.1	102.223	202.744	158.761	101.882	202.118	158.179
		0.1	102.223	158.322	202.200	101.882	157.804	201.640
		0.5	125.331	123.275	316.383	124.942	122.804	315.304
		0.9	233.370	102.853	705.482	232.914	102.599	704.089
5	3	–0.9	233.282	671.835	101.273	232.863	670.560	100.987
		–0.5	125.299	295.369	116.883	124.923	294.484	116.453
		–0.1	102.222	188.208	147.943	101.881	187.611	147.393
		0.1	102.222	147.583	187.752	101.881	147.085	187.209
		0.5	125.326	116.226	293.368	124.948	115.794	292.380
		0.9	233.358	100.984	668.075	232.989	100.778	667.051
5	5	–0.9	210.636	605.651	102.737	210.240	604.410	102.444
		–0.5	122.747	296.435	121.645	122.407	295.640	121.229
		–0.1	102.031	193.849	153.673	101.720	193.306	153.158
		0.1	102.031	153.272	193.352	101.720	152.816	192.869
		0.5	122.751	120.904	294.348	122.411	120.490	293.458
		0.9	210.584	102.365	602.036	210.246	102.156	601.060

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